Excercise 1 (Reaction systems)
Calculate the stoichiometric matrix for the following reaction systems consisting of chemical species and chemical elements. Determine then independent chemical reactions and their rate functions according to the mass action law.

1. \{(CO_2, H_2O, H_2CO_3), (C, H, O)\},
2. \{(N_2, O_2, H_2O, NO, NH_3, HNO_3), (H, N, O)\}.

Excercise 2 (Adsorption model)
Consider the adsorption model
\[
\begin{align*}
ct + st + Lc &= 0, \quad (1a) \\
s_t &= k_1c - k_2s. \quad (1b)
\end{align*}
\]
1. Derive an ODE for \(c\) by eliminating \(s\). To do so, reformulate (1a) and take the derivative with respect to time.
2. Use the solution ansatz \(c(t, x) = c_0 + f(t) \cos(nx)\) for \(Lc = -dc_{xx}\). Discuss the solutions behavior and compare it with a solution of the corresponding model without sorption.
3. Consider the case of local equilibrium for \(Lc = vc_x - dc_{xx}\). Eliminate again \(s\) and discuss the obtained result.

Excercise 3 (Adsorption model, traveling wave)
Consider the adsorption model
\[
\partial_t (c + \alpha \varphi(c)) + qc_x - dc_{xx} = 0. \quad (2)
\]
Investigate the „traveling wave“ solutions \(c(t, x) = u(x - vt)\) with boundary conditions \(c(t, \infty) = 0, c(t, -\infty) = u_l > 0\).
1. Derive an ODE of second order for \(u(z)\) with \(z = x - vt\). Integrate this intermediate result to obtain a first order ODE.
2. Derive appropriate boundary conditions for \(u\) and determine therewith the integration constant.
3. Show that \(v \leq q\) holds true.
4. Investigate the existence of „traveling wave“ solutions for Freundlich adsorption isotherms \(\varphi(u) = \sqrt{u}\). Use the substitution \(w := \sqrt{u}\) to solve the ODE in \(u\).
Excercise 4 (Stokes equations)

Consider the incompressible Stokes equations

\[-\mu \Delta v + \nabla p = 0,\]
\[\nabla \cdot v = 0.\]  

1. Consider a cylindrical pore of length $L$ with radius $R$ and surface $A$ of top and bottom. Derive the analytical solution (Poiseuille profile) of the Stokes equations in case of no-slip boundary conditions on the cylinder’s lateral surface and given pressure difference $\delta p = p_2 - p_1$ between cylinder’s top and bottom. Use cylinder coordinates.

2. Calculate the flux through one pore by integration over the cross section $A$.

3. Consider now a cylinder consisting of a pore network with $n$ parallel cylindrical pores and porosity $\theta = n\pi R^2 / A$. Determine the total flow trough the cylinder and discuss the results relation to Darcy’s law.

Excercise 5 (Upscaling, Darcy’s law)

Consider the $\varepsilon$-scales, incompressible Stokes-equations on a perforated domain $\bigcup Y_{\varepsilon} = \Omega_{\varepsilon} \subset \Omega = (0, L)^n$, $Y_1 \subset Y = (0, 1)^n$, $\partial \Omega_{\varepsilon} =: \Gamma_{\varepsilon}$.

\[-\varepsilon^2 \eta \Delta v_\varepsilon + \frac{1}{\rho} \nabla p_\varepsilon = 0 \quad x \in \Omega_{\varepsilon},\]  
\[\nabla \cdot v_\varepsilon = 0 \quad x \in \Omega_{\varepsilon},\]  
\[v_\varepsilon = 0 \quad x \in \Gamma_{\varepsilon}.\]  

Consider a formal asymptotic expansion

\[v_\varepsilon(x) = v_0(x, y) + \varepsilon v_1(x, y) + \varepsilon^2 v_2(x, y) + \ldots,\]
\[p_\varepsilon(x) = p_0(x, y) + \varepsilon p_1(x, y) + \varepsilon^2 p_2(x, y) + \ldots\]

with $y = x / \varepsilon$. Analyze the subproblems of order $\varepsilon^k$, $k \in \mathbb{Z}$ in order to derive Darcy’s Law

\[\bar{v}_0 := \int_{Y_1} v_0(x, y) \, dy = -\frac{K}{\rho \eta} \nabla p_0, \quad x \in \Omega\]  
\[\nabla \cdot \bar{v}_0 = 0 \quad x \in \Omega.\]  

1. Derive the relation $\nabla = \nabla_x + \frac{1}{\varepsilon} \nabla_y$.

2. Show that $p_0(x, y) = p_0(x)$ holds.
3. Deduce the following cell problems in \( w_j = (w_{ij})_{i=1,...,n}, \pi_j, j = 1, \ldots, n \) by linearity

\[
-\Delta w_j + \nabla \pi_j = -e_j, \quad y \in Y_l, \\
\nabla \cdot w_j = 0 \quad y \in Y_l, \\
w_j = 0 \quad y \in Y \setminus Y_l.
\]

\( w_j, \pi_j \) periodic

4. Integrate the equations of order \( \varepsilon^0 \) with respect to \( y \) and introduce the permeability tensor

\[
K_{ij} = \int_Y w_{ij} \, dy.
\]